Honors Pre-Calculus
Spring Semester Exam Prac
No Calculator

Name AR

For numbers 1 – 5, classify the conic section from its equation. Explain

1. $7x^2 - 6\sqrt{3}xy + 1y^2 - 16 = 0$

b^2-4ac
$(-6\sqrt{3})^2 - 4(7)(1) > 0$ hyperbola

2. $x^2 - 6xy + 9y^2 - 2y + 1 = 0$

$(-6)^2 - 4(1)(9) = 0$ parabola

3. $4x^2 + 6xy + 3y^2 - 6 = 0$

$(6)^2 - 4(4)(3) < 0$ ellipse a circle

4. $x^2 - y^2 + 16y - 128 = 0$

hyperbola $ac < 0$

5. $4x^2 + 4y^2 + 2x + 2y - 8 = 0$

circle $a = c$

6. Evaluate each of the following. When your answer is an angle, give the answer in both degrees and radians.

a. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{-\pi}{4}, -45^\circ$

b. arccos(-1) $\pi, 180^\circ$

c. $\sin\left(\tan^{-1}\left(-\frac{12}{5}\right)\right)$ $= \frac{-12}{13}$

5, 13, -12

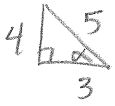
d. $\arctan\left(\tan\left(\frac{5\pi}{4}\right)\right)$ $\frac{\pi}{4}, 45^\circ$

1

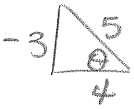
Use the following information to answer questions 7 and 8:

$\cos\alpha = \frac{3}{5},\ 0 \le \alpha \le \frac{\pi}{2}$ $\qquad$ $\sin\theta = -\frac{3}{5},\ \frac{3\pi}{2} \le \theta \le 2\pi$

QI



QIV



7. $\sin(2\alpha) =$ $2\sin\alpha\cos\alpha$

$2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \frac{24}{25}$

8. $\cos(\theta - \alpha) =$ $\cos\theta\cos\alpha + \sin\theta\sin\alpha$

$\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{-3}{5}\right)\left(\frac{4}{5}\right) = 0$

$\frac{\pi}{8}(x-5)$

9. For the function $y = 2 + 2\cos\left(\frac{\pi}{8}x - \frac{5\pi}{8}\right)$

   a. Graph at least one complete cycle

   b. Amplitude? 2

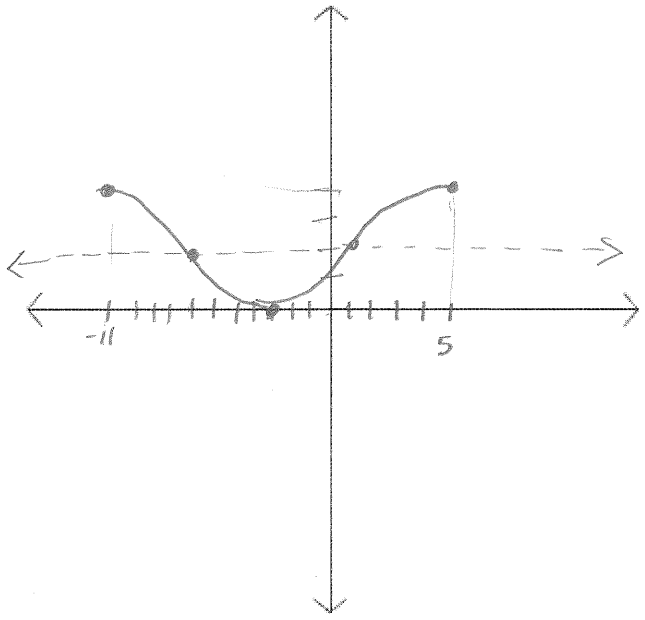
$\frac{2\pi}{\frac{\pi}{8}} = 16$ c. Period? 16

   d. Phase shift? right 5

   e. Vertical shift? up 2

   f. Maximum value of y? 4

   g. Minimum value of y? 0



10. If θ is in standard position with $270° < \theta < 360°$ and if $\sin\theta = q$, find an expression for each of the following in terms of q:

a. $\cot\theta$ $\frac{\sqrt{1-q^2}}{q}$

b. $\sec\theta$ $\frac{1}{\sqrt{1-q^2}}$

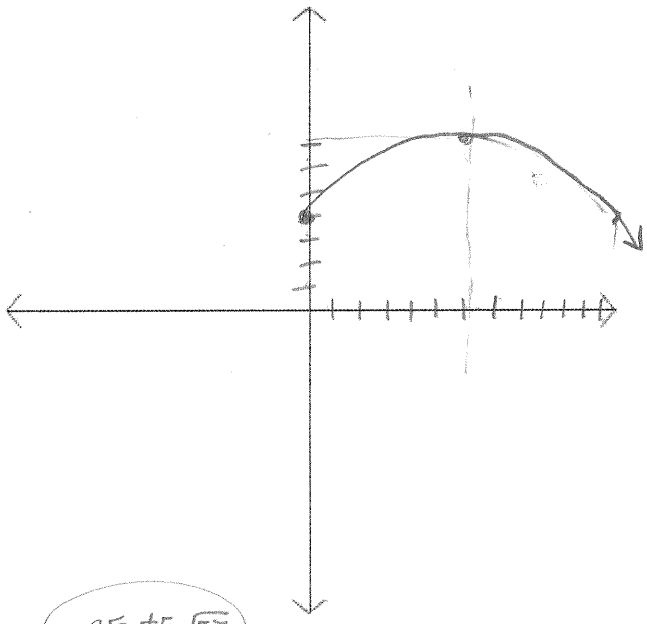
c. $\cos\theta$ $\sqrt{1-q^2}$

d. $\tan\theta$ $\frac{q}{\sqrt{1-q^2}}$

e. $\csc\theta$ $\frac{1}{q}$

11. The path of a softball is given by the equation $y = -0.08x^2 + x + 4$, where x and y are in feet, with x = 0 corresponding to the position where the ball was thrown.

a. Sketch the graph



b. What is the maximum height that the softball reaches? What is the horizontal distance that the ball has travelled at this point?

$V: \left(\frac{-b}{2a}\right)$

$\left(\frac{-1}{2(-.08)}\right)$

$\frac{-1}{-.16} = \frac{100}{16} = \frac{25}{4}$

max ht 7.125 ft

h. dist 6.25 ft

c. How far from the thrower is the softball when it hits the ground?

$0 = -.08x^2 + x + 4$

$2x^2 - 25x - 100 = 0$

$\frac{25 \pm \sqrt{625 - 4(2)(-100)}}{4}$ $\frac{25 \pm \sqrt{1425}}{4} =$ $\frac{25 + 5\sqrt{57}}{4}$

≈ 15.687

d. What are the coordinates of the focus of this parabola?

$4p(y-k) = (x-h)^2$ pt (0, 4)

$4p(y - 7.125) = (x - 6.25)^2$

$4p(4 - 7.125) = (-6.25)^2$ Focus (6.25, 4)

$p = \frac{(-6.25)^2}{4(-3.125)}$

$p = -3.125$

12. Prove the following identity: $\frac{(\cos x-1)(\cos x+1)}{\sin x} = -\sin x$

$\frac{\cos^2 x - 1}{\sin x}$

$\frac{-\sin^2 x}{\sin x}$

$-\sin x = -\sin x$

With Calculator Name _____

1. The positions of two particles traveling in straight lines are given by $s_1 = \sin t$ and $s_2 = \sin(t + \frac{\pi}{3})$, with s_1 and s_2 in meters and t in seconds. At what time(s) in the interval $[0, 2\pi)$ do the particles meet?

$\sin t = \sin\left(t + \frac{\pi}{3}\right)$

$\sin t = \sin t \cos\left(\frac{\pi}{3}\right) + \cos t \sin\left(\frac{\pi}{3}\right)$

$\sin t = \sin t \left(\frac{1}{2}\right) + \cos t \left(\frac{\sqrt{3}}{2}\right)$

$2\sin t = \sin t + \sqrt{3}\cos t$

$\sin t = \sqrt{3}\cos t$

$\sin^2 t = 3\cos^2 t$

$1 - \cos^2 t = 3\cos^2 t$

$1 = 4\cos^2 t$

$\frac{1}{4} = \cos^2 t$

$\pm\frac{1}{2} = \cos t$

$\rightarrow t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

2. A job has a starting salary of \$34 000 and a guaranteed salary increase of \$2250 per year for the first four years of employment.
   a. What are the salaries for the first 5 years of employment?

\$34,000, \$36,250, \$38,500, \$40750, \$43000

   b. Use sigma notation to write an expression for the total amount of money earned in the first 5 years.

$\sum_{i=1}^{5}(34{,}000 + 2250(i-1)) = \sum_{i=1}^{5}(31750 + 2250i)$

   c. What is the total amount of money earned in the first 5 years?

\$192,500

3. For the series $20 + 8 + \frac{16}{5} + \cdots$
   a. Find the 7th term

$r = \frac{2}{5}$

$a_n = 20\left(\frac{2}{5}\right)^{n-1}$ $a_7 = 20\left(\frac{2}{5}\right)^6 = \frac{256}{3125}$

   b. Find the 7th partial sum

$\sum_{n=1}^{7}\left[20\left(\frac{2}{5}\right)^{n-1}\right] = 20\left(\frac{1-\left(\frac{2}{5}\right)^7}{1-\left(\frac{2}{5}\right)}\right) = \frac{103996}{3125}$

   c. Find the sum of the infinite series

$S = \frac{20}{1-\frac{2}{5}} = \frac{20}{3/5} = \frac{100}{3}$

4. For the sequence 112, 96, 80, …
   a. Find the 82nd term

$d = -16$

$a_n = 112 - 16(n-1)$

$a_{82} = -1184$

   b. Find the sum of the first 82 terms.

$\frac{82}{2}(112 - 1184) = -43952$

5. In his first trip baling hay around a field, a farmer makes 123 bales. In his second trip, he makes 11 fewer bales. If this pattern continues, how many total bales will the farmer make if he makes another 6 trips around the field?

$123, 112, \ldots$ $\quad a_8 = 123 - 77 = 46$

$$\sum_{n=1}^{8}[123-11(n-1)] = \frac{8}{2}(123+46)$$
$$= 4(169)$$
$$= 676$$

6. Express $2.6\overline{7}$ as a fraction in lowest terms.

$2.6\overline{7} = 2.6 + .07 + .007 + \ldots$

$$2.6 + \frac{.07}{1-\frac{1}{10}} = 2.6 + \frac{\frac{7}{100}}{\frac{9}{10}} = \frac{26}{10} + \frac{7}{90}$$
$$= \frac{234+7}{90} = \frac{241}{90}$$

7. Prove by mathematical induction: $3 + 8 + 13 + 18 + \cdots + (5n - 2) = \frac{n}{2}(5n + 1)$

$n=1$: $3 = \frac{1}{2}(5+1)$ ✓

Assume for $n=k$ $\quad 3+8+13+\ldots+(5k-2) = \frac{k}{2}(5k+1)$

Show for $n=k+1$ $\quad \boxed{3+8+\ldots+(5k-2)}+(5k+3) = \frac{k+1}{2}(5(k+1)+1) = \frac{k+1}{2}(5k+6)$

$$\frac{k}{2}(5k+1) + (5k+3)$$
$$\frac{5k^2+k+10k+6}{2}$$
$$\frac{5k^2+11k+6}{2}$$
$$\frac{(5k+6)(k+1)}{2}$$

So $3+8+\ldots+(5k+3) = \frac{k+1}{2}(5k+6)$

Thus by PMI $3+8+\ldots+(5n-2) = \frac{n}{2}(5n+1)$

8. Write an equation for an ellipse with center in quadrant I and with its major axis parallel to the x-axis. Give the coordinates of the foci for your ellipse. Use the eccentricity to verify that you have an ellipse. Give any other description of your ellipse.

Ans will vary